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EB Education Revision Guide

How to work with Algebra: Part 2

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What you need to know about Algebra: Part 2

Including:

- Powers and Roots
- Expanding Brackets:

  Single

  Double

  Triple

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Powers and roots

How to do it:

You need to remember these rules:

- When multiplying you add the powers.
- When dividing you subtract the powers.
- When raising one power to another you multiply them.
- Anything to the power of 1 is itself.

Example:

- $m^2 \times m^6 = m^8$
- $m^9 \div m^2 = m^7$
- $(m^3)^2 = m^{3 \times 2} = m^6$
- $y^1 = y$

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Powers and roots

How to do it:

- Anything to the power of 0 is 1.
- 1 to any power is still just 1.
- In fractions you need to apply the power to the top and the bottom of the fraction.

Example 1:

- $y^0 = 1$ $k^0 = 1$
- $1^8 = 1$ $1^{93} = 1$
- $(\frac{h}{r})^3 = \frac{h^3}{r^3}$

Powers and roots

How to do it:

These rules are a bit trickier:

- Negative powers – turn it upside down and make the power positive.
- Turning a number upside down is called 'taking a recipricol'.

Example 2:

5^{-2}

TOP TIP: Remember that 5 can be written as $\frac{5}{1}$

$\frac{1}{5^2} = \frac{1}{25}$

Do the same if it is letters

$b^{-3} = \frac{1}{b^3}$

$(\frac{4}{6})^{-2}$ **Turn it upside down**

Remember the power applies to the top and the bottom

$\frac{6^2}{4^2} = \frac{36}{16}$

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Powers and roots

How to do it:

Fractional powers:

$\frac{1}{2}$ means Square Root

$\frac{1}{3}$ means Cube Root

$\frac{1}{4}$ means Fourth Root

If the power is a negative fraction – you need to remember to turn it upside down AND complete the root.

TOP TIP: Learn your square numbers and square roots up to at least 15, and your cube numbers and cubic roots up to 10

Example 3:

$36^{\frac{1}{2}} = \sqrt{36} = 6$

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$

$y^{\frac{1}{5}} = \sqrt[5]{y}$

$49^{-\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{49}} = \frac{1}{7}$

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Powers and roots

How to do it:

Two-stage fractional powers:

You need to split the fraction into a root and a power.

You do the root first.

Then you do the power.

Example :

$$(64)^{\frac{5}{6}}$$

This means

$(64)^{\frac{1}{6} \times 5}$ $\frac{1}{6}$ **is the root**

5 is the power

Top Tip – this is the same as taking the square root 3 times

Root first $\sqrt[6]{64} = 2$

Power second $(2)^5 = 32$

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Powers and roots

Example:

Simplify $(4a^2b^5e^2f)^3$

Complete each term separately

$(4)^3$ =	4 x 4 x 4	= 64
$(a^2)^3$=	$a^{2 \times 3}$	= a^6
$(b^5)^3$=	$b^{5 \times 3}$	= b^{15}
$(e^2)^3$=	$e^{2 \times 3}$	= e^6
f^3		

Put them all together

$= 64a^6b^{15}e^6f^3$

Expanding brackets: Single brackets

How to do it:

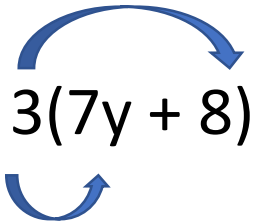
Expanding brackets means multiplying them out.

You need to remember that anything outside the brackets is multiplied by EVERY term inside the brackets.

You do not put a x sign in between letters when you multiply them. For example m x t = mt, p x p = p^2.

You need to remember the rules for multiplying – and + terms.

Example 1:



(3 x 7y) + (3 x 8)

= 21y + 24

Expanding brackets: Single brackets

Example 2:

$3m\,(t - 4y)$

Remember + multiplied by - makes a -

$(3m \times t) + (3m \times -4y)$

$= 3mt - 12my$

Example 3:

$-6(4p^2 - 3q^3)$

Remember - multiplied by – makes a +

$(-6 \times 4p^2) + (-6 \times -3q^3)$

$= -24p^2 + 18q^3$

Expanding brackets: Double brackets

How to do it:

You need to multiply everything in the first bracket by everything in the second bracket.

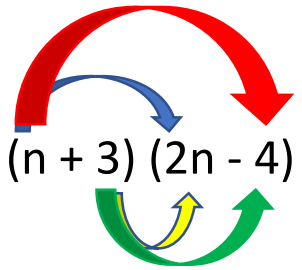
There are 2 ways to multiply out double brackets

- the FOIL method
- the GRID method

Example 1: FOIL method

Expand and simplify

(n + 3) (2n-4)



FIRST $n \times 2n = 2n^2$

OUTSIDE $n \times -4 = -4n$

INSIDE $3 \times 2n = 6n$

LAST $3 \times -4 = -12$

Collect like terms
$-4n + 6n = 2n$

$= 2n^2 + 2n - 12$

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Expanding brackets: Double brackets

Example 2: GRID Method

Expand and simplify

$(3w – 6)(4w – 5)$

x	3w	-6
4w	$12w^2$	-24w
-5	-15w	+30

Collect like terms -15w – 24w = -39w

$= 12w^2 - 39w + 30$

Example 3:

Expand and simplify

$(4y + 8)^2$

You need to write out the brackets in full - do not just square each term!!

(4y+8) (4y + 8)

$16y^2 + 32y + 32y + 64$

$= 16y^2 + 64y + 64$

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Expanding brackets: Double brackets

Example 4:

Expand and simplify

$(3p + 6)^2$

You need to write out the brackets in full - do not just square each term!!

$(3p + 6)(3p + 6)$

$9p^2 + 18p + 18p + 36$

$= 9p^2 + 36p + 36$

Example 5:

Expand and simplify

$(5w + 9)^2$

You need to write out the brackets in full - do not just square each term!!

$(5w + 9)(5w + 9)$

$25w^2 + 45w + 45w + 81$

$= 25w^2 + 90w + 81$

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Expanding brackets: Triple brackets

How to do it:

You just need to multiply 2 brackets together first, then multiply the answer to this by the remaining bracket.

FOIL does not work with triple brackets as there are too many terms so you may want to use the grid method for these.

Example:

$(x + 2)(x + 4)(3x - 2)$

Multiply out 2 brackets. You can use FOIL or GRID method.

$(x + 2)(x + 4) = x^2 + 6x + 8$

Multiply the answer by the remaining bracket.

	x^2	$+ 6x$	$+ 8$
$+3x$	$3x^3$	$18x^2$	$24x$
-2	$-2x^2$	$-12x$	-16

Collect like terms.

$-2x^2 + 18x^2 = 16x^2$ $\quad$ $-12x + 24x = 12x$

Complete the answer

$= 3x^3 + 16x^2 + 12x - 16$

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Expanding brackets: Triple brackets

Example 2:

$(2x + 3)^2 (x - 1)$

Multiply out 2 brackets

$(2x + 3)^2 = (2x + 3)(2x + 3) = 4x^2 + 12x + 9$

Multiply the answer by the remaining bracket

	$+4x^2$	$+12x$	$+9$
$+x$	$4x^3$	$12x^2$	$9x$
-1	$-4x^2$	$-12x$	-9

Collect like terms

$-4x^2 + 12x^2 = 8x^2$ $\quad$ $-12x + 9x = -3x$

Complete the answer

$= 4x^3 + 8x^2 - 3x - 9$

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Your turn:

Set 1:

1. $6 \times 6 \times 6 \times 6 \times 6$ as a power of 6
2. $3^2 \times 3^6$ as a power of 3
3. $2^6 \div 2^4$ as a power of 2
4. 10^0
5. 10^{-2}

Set 2:

1. Find n $280 = 2^n \times 5 \times 7$
2. $(9^7)^2$
3. Find n $\dfrac{2^9 \times 2^4}{2^n} = 2^8$
4. Find n $\dfrac{5^n}{5^4 \times 5^6} = 5^3$
5. 2^{-3}
6. $\left(\frac{27}{343}\right)^{\frac{1}{3}}$

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Your turn:

Set 3:

1. $5\,(3d + 8)$
2. $7\,(4a - 4)$
3. $-3(2p^2 - 4q^3)$
4. $(n + 7)\,(2n + 3)$
5. $(2n + 5)\,(2n - 8)$
6. $(p - 7)\,(2p - 8)$

Set 4:

1. $(3d + 7)^2$
2. $(x + 2)(x + 3)(x + 6)$
3. $(2y + 3)(y + 1)(3y - 1)$
4. $(y + 3)(y + 1)^2$
5. A cube has side length $x + 5$ cm. Show that the volume of the cube is $x^3 + 15x^2 + 75x + 125$

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Answers:

Set 1:

1. 6^5
2. 3^8
3. 2^2
4. 1
5. $\frac{1}{10^2} = \frac{1}{100}$

Set 2:

1. $2^n = \frac{280}{35}$ $2^n = 8$ $n = 3$
2. 9^{14}
3. $9 + 4 - n = 8$

   $n = 13 - 8$ $n = 5$
4. $\frac{5^n}{5^{10}} = 5^3$ $n - 10 = 3$ $n = 13$
5. $\frac{1}{8}$
6. $\frac{\sqrt[3]{27}}{\sqrt[3]{343}} = \frac{3}{7}$

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Answers:

Set 3:

1. $15d + 40$
2. $28a - 28$
3. $-6p^2 + 12q^3$
4. $2n^2 + 17n + 21$
5. $4n^2 - 6n - 40$
6. $2p^2 - 22p + 56$

Set 4:

1. $9d^2 + 42d + 49$
2. $x^3 + 11x^2 + 36x + 36$
3. $6y^3 + 13y^2 + 4y - 3$
4. $y^3 + 5y^2 + 7y + 3$
5. $(x + 5)^2 = x^2 + 10x + 25$

   $(x^2 + 10x + 25)(x + 5)$

   $= x^3 + 15x^2 + 75x + 125$

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For more help and resources, or to work with us as a tutor, please contact us

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