

# EB Education Revision Guide



How to work with Fractions

# What you need to know about fractions

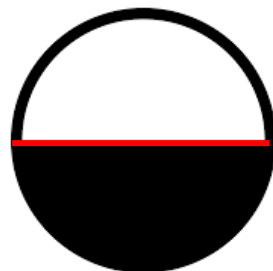
- Equivalent fractions
- Simplifying fractions
- Improper fractions and mixed numbers
- Finding fractions of amounts
- Adding and subtracting fractions with same denominator

- Adding and subtracting fractions with different denominators
- Multiplying and dividing fractions
- Ordering fractions with different denominators
- Fractions and decimals
- Fractions and percentages

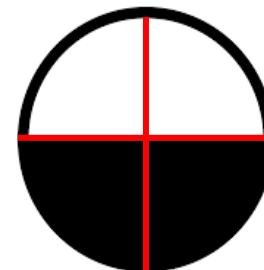
- Recurring decimals
- Simplifying algebraic fractions
- Adding and subtracting algebraic fractions
- Multiplying and dividing algebraic fractions
- Solving algebraic fractions

# Equivalent Fractions

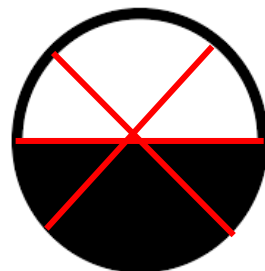
These may look different,  
but they are the same  
amount OR equivalent.



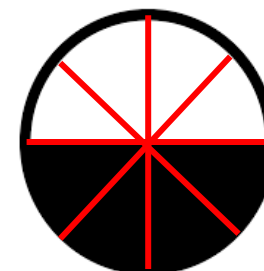
$$\frac{1}{2}$$



$$\frac{2}{4}$$



$$\frac{3}{6}$$



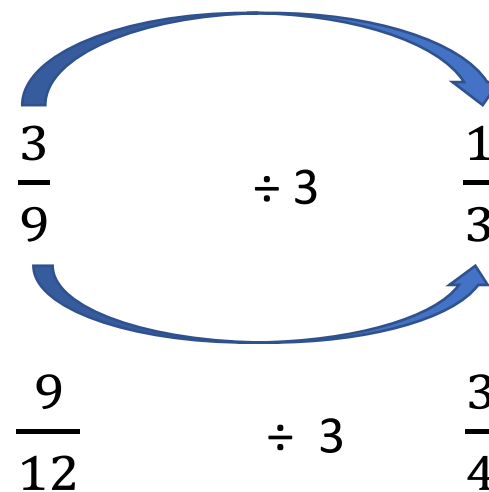
$$\frac{4}{8}$$

# To find out if fractions are equivalent:

## How to do it:

- Simplify the fraction – by finding the biggest factor (number) that will go into the numerator (top number) and denominator (bottom number).
- Divide by that number.

## Example:



The diagram illustrates the simplification of two fractions. For the first fraction,  $\frac{3}{9}$ , a blue arrow curves from the numerator 3 to the denominator 9, and another blue arrow curves from the denominator 9 back to the numerator 3, with a  $\div 3$  in the center. This results in the simplified fraction  $\frac{1}{3}$ . For the second fraction,  $\frac{9}{12}$ , a blue arrow curves from the numerator 9 to the denominator 12, and another blue arrow curves from the denominator 12 back to the numerator 9, with a  $\div 3$  in the center. This results in the simplified fraction  $\frac{3}{4}$ .

# Simplifying Fractions

## How to do it

You should always cancel down fractions or SIMPLIFY as much as possible when answering questions.

To do this try to find the highest common factor of the top and bottom number (numerator and denominator) and divide by this number.

When you cannot divide the numbers anymore it is fully simplified.

## Example

$$\frac{15}{35} = \frac{3}{7}$$



$$\frac{16}{136} = \frac{2}{17} \quad \text{Divide by 8}$$

# Mixed numbers and Improper Fractions

## Mixed Numbers

A fraction with a whole number in front of it is known as a mixed number.

$7 \frac{3}{5}$  means 7 and 3 fifths

## Improper Fractions

A fraction where the numerator is bigger than the denominator is known as an improper fraction or top heavy fraction.

$$\frac{9}{4} \quad \frac{17}{3}$$

You need to be able to convert mixed numbers into improper fractions and change improper fractions into mixed numbers.

# Improper to mixed number

## How to do it:

Work out how many whole numbers there are by dividing the numerator by the denominator.

The remainder will be the new numerator.

Leave the denominator as it was.

## Example:

$$\frac{17}{3}$$

$$17 \div 3 = 5 \text{ remainder } 2$$


$$5 \frac{2}{3}$$

# Mixed number to improper

## How to do it:

- Look at the **denominator** of the fraction part first. This will be the denominator of the improper fraction.
- Multiply the denominator by the whole number.
- Add the numerator.
- Put this over the original denominator.

## Example:

$$3\frac{7}{8}$$


$$3 \times 8 = 24$$

Add 7

$$24 + 7 = 31$$

$$\frac{31}{8}$$



# Adding and subtracting

## How to do it:

They **MUST** have the same denominator.

Fractions with the same denominator can be added or subtracted – by simply adding or subtracting the numerator.

## Example:

$$\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$$

$$\frac{9}{11} - \frac{3}{11} = \frac{6}{11}$$

# Adding and subtracting

## How to do it:

If they have a different denominator then you need to find a common denominator by making equivalent fractions.

Find the lowest common multiple of both denominators (need a number which is in the times table of both).

Write the equivalent fractions using the same denominator.

Add or subtract the fraction.

### Example 1:

$$\frac{2}{3} + \frac{1}{4}$$

We need a number in the 3 and 4 times table to be the new denominator  
We can use 12

The denominator is multiplied by 4 to make 12, so the top must be as well

The denominator is multiplied by 3 to make 12, so the top must be as well

$$\frac{2}{3} = \frac{8}{12}$$

$$\frac{1}{4} = \frac{3}{12}$$

$$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$$

Now you can add the fractions together.

# Adding and subtracting

## How to do it:

If they have a different denominator then you need to find a common denominator by making equivalent fractions.

Find the lowest common multiple of both denominators (need a number which is in the times table of both).

Write the equivalent fractions using the same denominator.

Add or subtract the fraction.

## Example 2:

$$\frac{4}{6} - \frac{2}{7}$$

We need a number in the 6 and 7 times table to be the new denominator. We can use 42

The denominator is multiplied by 7 to make 42, so the top must be as well

The denominator is multiplied by 6 to make 42, so the top must be as well

$$\frac{4}{6} = \frac{28}{42}$$

$$\frac{2}{7} = \frac{12}{42}$$

$$\frac{28}{42} - \frac{12}{42} = \frac{16}{42} = \frac{8}{21}$$

Now you can subtract the fractions and simplify.

# Adding and subtracting mixed numbers

## How to do it:

If the sum contains a mixed number, you should first change it into an improper fraction.

Then follow the same method.

### Example 3:

$$3\frac{3}{5} - \frac{3}{4} = 3\frac{3}{5} = \frac{18}{5}$$

We need a number in the 5 and 4 times table to be the new denominator. We can use 20.

$$\frac{18}{5} - \frac{3}{4}$$

$$\frac{18}{5} = \frac{72}{20}$$

The denominator is multiplied by 5 to make 20, so the top must be as well

$$\frac{3}{4} = \frac{15}{20}$$

The denominator is multiplied by 4 to make 20, so the top must be as well.

$$\frac{72}{20} - \frac{15}{20} = \frac{57}{20} = 2\frac{17}{20}$$

Now you can subtract the fractions and change to a mixed number.

# Multiplying

## How to do it:

Multiplying fractions is simple.  
All you need to do is times the numbers at the top, and times the numbers at the bottom.

Check your answer to see if it can be simplified.

If the fractions contain mixed numbers - change them to improper fractions before multiplying.

## Example 1:

$$\frac{2}{3} \times \frac{1}{4} = \frac{\cancel{2}}{12}$$

$$\frac{1}{6}$$

To simplify, divide top and bottom by 2

$$\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$$

# Multiplying

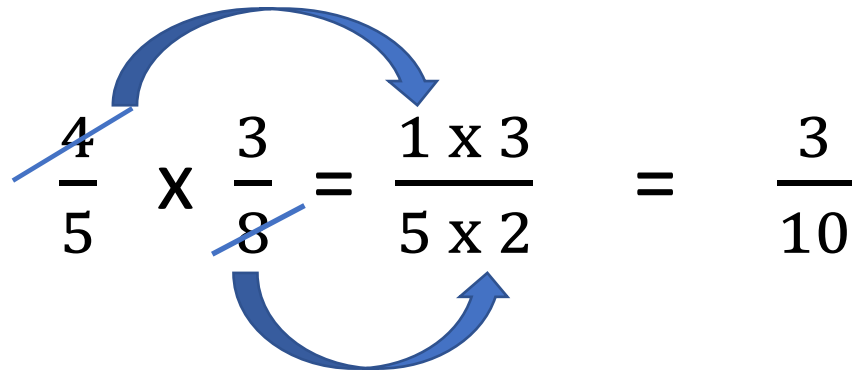
## How to do it:

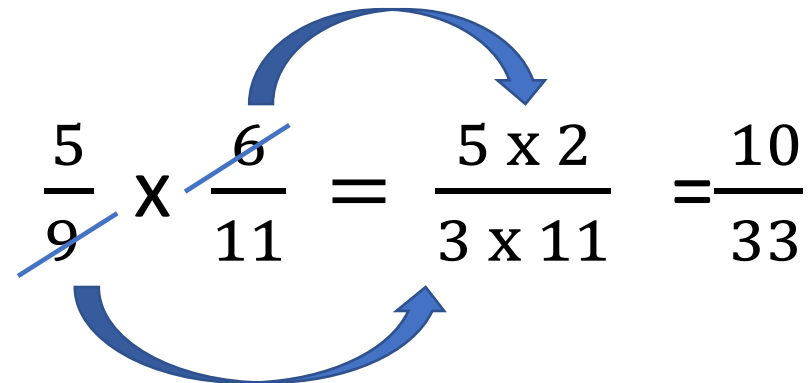
Sometimes you can simplify before multiplying.

To simplify, divide 4 and 8 by 4

To simplify, divide 9 and 6 by 3

## Example 2:

$$\frac{4}{5} \times \frac{3}{8} = \frac{1 \times 3}{5 \times 2} = \frac{3}{10}$$


$$\frac{5}{9} \times \frac{6}{11} = \frac{5 \times 2}{3 \times 11} = \frac{10}{33}$$


# Dividing

## How to do it:

To divide you need to:

**KEEP** the first fraction.

**CHANGE** the  $\div$  to  $\times$

**FLIP** the second fraction.

Then just follow the rules for multiplying fractions.

If there are mixed numbers – change to improper fractions first.

## Example:

$$\frac{4}{5} \div \frac{3}{7}$$

**KEEP CHANGE FLIP**

$$\frac{4}{5} \times \frac{7}{3} = \frac{28}{15} = 1\frac{13}{15}$$

# Finding fractions of amounts

## How to do it:

Remember this rule.

Divide by the bottom.

Times by the top.

## Example :

Find  $\frac{2}{5}$  of £75

**Step 1: Divide £75 by 5**

(bottom of fraction or denominator) = 15

**Step 2: Times 15 by 2**

(top of fraction or numerator) = £30



# Ordering fractions

## How to do it:

If you are asked to put fractions into ascending or descending order – you need to change them into equivalent fractions first.

6, 10, 5, 2 and 15 are all factors of 30

Put in order of size from smallest to largest

Rewrite in order using the ORIGINAL fractions.

## Example :

Put the following into ascending order

$$\frac{2}{6} \quad \frac{6}{10} \quad \frac{4}{5} \quad \frac{1}{2} \quad \frac{7}{15}$$

Make 30 the common denominator

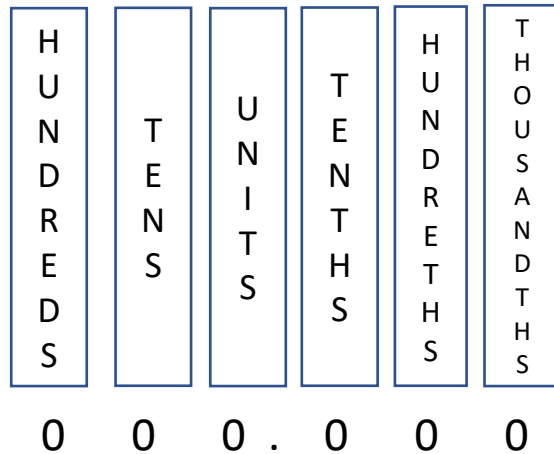
<b>x5</b>	<b>x3</b>	<b>x6</b>	<b>x15</b>	<b>x2</b>
$\frac{10}{30}$	$\frac{18}{30}$	$\frac{24}{30}$	$\frac{15}{30}$	$\frac{14}{30}$

1st      4th      5th      3rd      2nd

$\frac{2}{6}$	$\frac{7}{15}$	$\frac{1}{2}$	$\frac{6}{10}$	$\frac{4}{5}$
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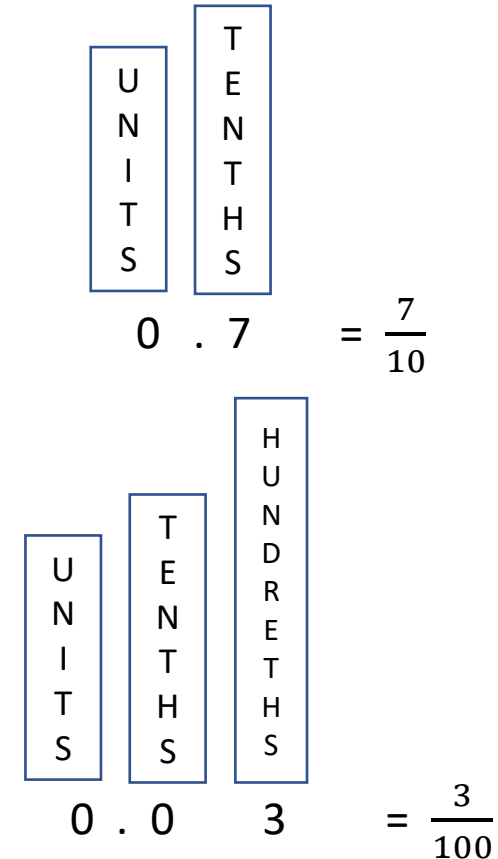
# Changing terminating decimals to fractions

## How to do it:



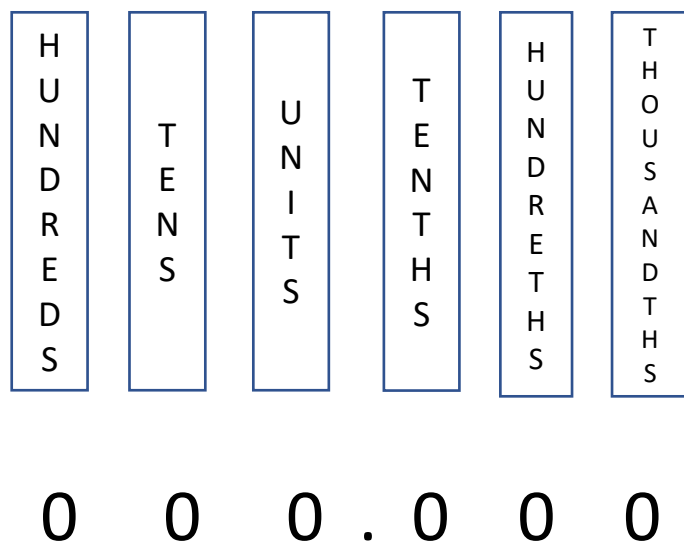
You can use place value to change from fractions into decimals. The digits after the decimal point go on the top of the fraction, and a power of 10 goes on the bottom – with the same number of zeros as there were decimal places.

## Example:



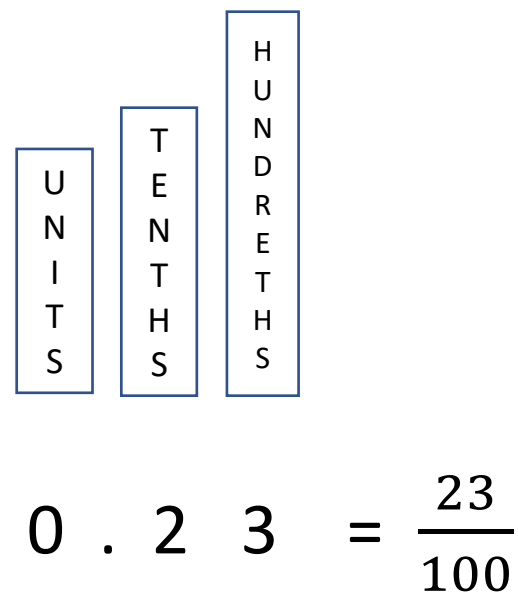
# Changing terminating decimals to fractions

## How to do it:



You can use place value to change from decimals into fractions.

## Example:



# Changing recurring decimals to fractions

## How to do it:

Recurring decimals have a pattern of numbers which keep repeating.

It is not always the same number which repeats – it can be a pattern of numbers.

You can tell how many numbers are repeated by how many numbers have a dot on top. If there is only 1 dot – just that number is repeated. If there are 2 dots, everything from the first to the second dot is repeated.

## Example:

$$\frac{1}{3} = 0.333333333$$

$$0.3\dot{4} = 0.3444444$$

$$0.\dot{3} \dot{4} = 0.3434343434$$

$$0.\dot{3}4\dot{7} = 0.347347347$$

# Recurring decimals

## How to do it:

You can use the denominator of a fraction (when it is simplified) to work out if it will convert to a recurring decimal or a terminating decimal.

If the fraction only has prime factors of 2 and 5 – it will be a terminating decimal.

ALL other fractions will be recurring decimals.

## Example

Fraction	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{20}$
Prime factors	5	2	2	2 & 5
Decimal	0.2	0.25	0.125	0.05

Fraction	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{14}$
Prime factors	3	2 & 3	2 & 7
Decimal	$0.\dot{3}$	$0.1\dot{6}$	$0.0\dot{7}1428\dot{5}$

# Recurring decimals into fractions

## How to do it:

To change a recurring decimal into a fraction.

Give your decimal number a name, like  $d$  or  $r$ , for example to turn it in to algebra.

Multiply the decimal number by a power of 10 to get the whole of the repeating part past the decimal point.

Subtract so you can get rid of the decimal.

Rearrange the algebra to get a fraction. Simplify your fraction.

## Example 1:

$$0.\dot{2}5\dot{5}$$

$$d = 0.\dot{2}5\dot{5} \times 1000$$

$$1000d = 255.\dot{2}5\dot{5}$$

$$d = 0.\dot{2}5\dot{5}$$

$$999d = 255$$

Subtract  $d$  from  
 $1000d$

$$d = \frac{255}{999} = \frac{85}{333}$$

# Recurring decimals into fractions

## How to do it:

If the recurring part of the decimal does not come straight after the decimal point.

Give your decimal a name.

Multiply the decimal by a power of 10 to get the non-repeating part past the decimal point.

Multiply again to get a full repeated part past the decimal point.

Subtract to get rid of the decimal part and follow the remaining steps as before.

## Example 2:

$$0.2\dot{5}$$

$$d = 0.2\dot{5} \quad \times 10$$

$$10d = 2.\dot{5}$$

$$100d = 25.\dot{5}$$

$$10d = 2.\dot{5}$$

$$90d = 23$$

Subtract 10d from 100d

$$d = \frac{23}{90}$$

$$90$$

# Changing fractions to decimals

## How to do it:

To change a fraction to a decimal – you need to divide the top by the bottom.

Use 'bus stop' division to help.

## Example:

$$\frac{7}{20} \text{ is } 7 \div 20$$

$$= 0.35$$



# Fractions into recurring decimals

## How to do it:

You can either

- Find an equivalent fraction with all nines on the bottom.
- The number on top is the recurring part of the decimal.
- The number of nines on the bottom tells you how many digits are in the recurring part of the fraction.

OR

- Divide the fraction.

## Example:

$$\frac{7}{33} = \frac{21}{99}$$

Multiply top and bottom by 3

$$\frac{21}{99} = 0.\dot{2}\dot{1}$$

If it was  $\frac{21}{999} = 0.\dot{0}2\dot{1}$

# Algebraic fractions: Simplifying

## How to do it:

You need to cancel terms on the top and the bottom.

Simplify numbers, and each letter separately.

## Example 1:

$$\frac{21 w^3 y^2}{14 w y^3}$$

$$\frac{21}{14} \div 7 = \frac{3}{2}$$

$$\frac{w^3}{w} \div w = w^2 \quad \text{left on top}$$

$$\frac{y^2}{y^3} \div y^2 = y \quad \text{left on bottom}$$

$$\frac{21 w^3 y^2}{14 w y^3} = \frac{3w^2}{2y}$$

# Algebraic fractions Simplifying.

## How to do it:

Sometimes you will need to factorise before you can simplify.

## Example 2:

$$\frac{3x + 6}{3x^2}$$

$$\frac{\cancel{3}(x + 2)}{\cancel{3}x^2}$$

$$\frac{x + 2}{x^2}$$

# Algebraic fractions: Simplifying

## How to do it:

Factorise the top expression by difference of 2 squares.

Factorise the quadratic on the bottom.

Cancel out the common factor.

## Example 5:

$$\frac{x^2 - 144}{x^2 + 7x - 60}$$

$$\frac{(\cancel{x+12})(x-12)}{(\cancel{x+12})(x-5)}$$

$$\frac{x-12}{x-5}$$

# Algebraic fractions: Adding and subtracting

## How to do it:

Work out the common denominator.

Multiply the top and the bottom of each fraction by whatever gives you the common denominator.

Add or subtract the numerators.

## Example:

$$\frac{7x}{3} + \frac{3x}{4}$$

Common denominator is 12

$$\frac{7x}{3} = \frac{28x}{12}$$

Multiply top and bottom by 4

$$\frac{3x}{4} = \frac{9x}{12}$$

Multiply top and bottom by 3

$$\frac{28x}{12} + \frac{9x}{12} = \frac{37x}{12}$$

# Algebraic fractions: Adding and subtracting

## Example 2:

$$\frac{25}{3x} - \frac{7}{x}$$

Common denominator is  $3x$

$$\frac{25}{3x} - \frac{21}{3x}$$

Multiply 7 and  $x$  by 3

$$\frac{4}{3x}$$

Complete the sum

## Example 3:

$$\frac{7}{x+4} - \frac{9}{x-2}$$

Common denominator is  $(x+4)(x-2)$

$$\frac{7(x-2) - 9(x+4)}{(x+4)(x-2)}$$

Multiply Fraction 1 by  $(x-2)$  and Fraction 2 by  $(x+4)$

$$\frac{7x-14 - 9x-36}{(x+4)(x-2)}$$

Expand the top brackets

$$\frac{-2x - 50}{(x+4)(x-2)}$$

# Algebraic fractions: Adding and subtracting

## Example 4:

$$\frac{x+3}{6} + \frac{x-2}{4}$$

Common denominator is 24

Multiply fraction 1 by 4, and fraction 2 by 6

$$\frac{4(x+3) + 6(x-2)}{24}$$

Expand the top brackets

$$\frac{4x+12 + 6x-12}{24} = \frac{10x}{24}$$

## Example 5:

$$\frac{3}{x^2+7x-30} + \frac{5}{x^2-100}$$

Factorise the expressions

$$\frac{3}{(x+10)(x-3)} + \frac{5}{(x+10)(x-10)}$$

Find the common denominator

$$\frac{3(x-10)}{(x+10)(x-3)(x-10)} + \frac{5(x-3)}{(x+10)(x-3)(x-10)}$$

Multiply to get common denominator

$$\frac{3x-30 + 5x-15}{(x+10)(x-3)(x-10)}$$

Expand the top brackets

# Algebraic fractions: Multiplying

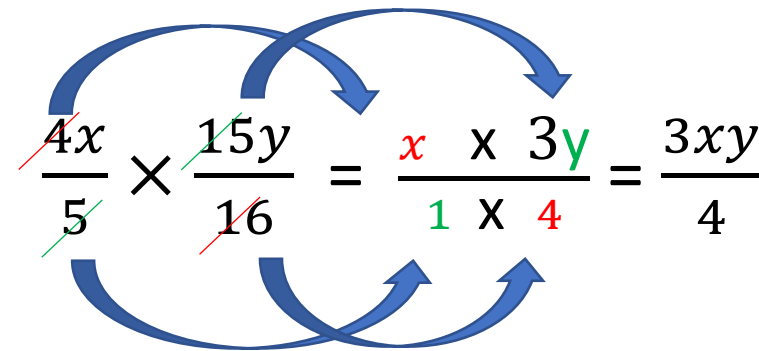
## How to do it:

Follow the same method as multiplying numerical fractions.

Multiply the tops and bottoms separately.

If you can, simplify before multiplying.

## Example:


$$\frac{\cancel{4}x}{\cancel{5}} \times \frac{1\cancel{5}y}{\cancel{4}16} = \frac{x \times 3y}{1 \times 4} = \frac{3xy}{4}$$

To simplify:

Divide **4x** and **16** by 4

Divide **5** and **15y** by 5

Multiply the top together

Multiply the bottom together.





# Algebraic fractions: Multiplying

**Example 2:**

$$\frac{x + 4}{x - 3} \times \frac{3x - 9}{5x + 20}$$

Factorise the expressions.

$$\frac{\cancel{x + 4}}{\cancel{x - 3}} \times \frac{3(\cancel{x - 3})}{5(\cancel{x + 4})}$$

Simplify.

$$\frac{3}{5}$$

# Algebraic fractions: Dividing

## Example 1:

Follow the same rules as dividing numerical fractions.

**KEEP** the first fraction

**CHANGE** the sign from divide to multiply

**FLIP** the second fraction

Complete the multiplication.

How to do it:

$$\frac{x}{3} \quad \div \quad \frac{2x}{7}$$

**KEEP CHANGE FLIP**

$$\frac{x}{3} \quad \times \quad \frac{7}{2x} = \frac{7x}{6x}$$

# Algebraic fractions

## Dividing

### Example 2

$$\frac{x^2-4}{20y^4} \div \frac{3x+6}{16y^2}$$

<b>KEEP</b>	<b>FLIP</b>	<b>CHANGE</b>
$\frac{x^2-4}{20y^4}$	X	$\frac{16y^2}{3x+6}$

Factorise and cancel

$$\frac{\cancel{(x+2)}(x-2)}{20\cancel{y^4}} \times \frac{\cancel{16y^2}}{3(x+2)}$$

Divide  $20y^4$  and  $16y^2$  by  $4y^2$

Cancel out  $x+2$  on both sides

$$\frac{x-2}{5y^2} \times \frac{4}{3} = \frac{4(x-2)}{15y^2}$$

### Example 3:

$$\frac{x^2-4}{x^2+4x-32} \div \frac{x^2-5x+6}{5x+40}$$

<b>KEEP</b>	<b>FLIP</b>	<b>CHANGE</b>
$\frac{x^2-4}{x^2+4x-32}$	$\div$	$\frac{5x+40}{x^2-5x+6}$

Factorise and cancel

$$\frac{(x+2)\cancel{(x-2)}}{\cancel{(x+8)}(x-4)} \times \frac{5\cancel{(x+8)}}{(x-3)\cancel{(x-2)}}$$

$$\frac{5(x+2)}{(x-4)(x-3)}$$

$$\frac{5x+10}{(x-4)(x-3)}$$

# Algebraic Fractions: Solving

## How to do it:

Remove the fractions.

Collect the letter terms on one side,  
and the number terms on the other.

Solve the equation.

## Example 1:

$$\frac{5a+3}{4} = 7$$

x 4 both sides

$$5a + 3 = 28$$

- 3 both sides

$$5a = 25$$

÷ 5 both sides

$$a = 5$$

# Algebraic fractions: Solving

## How to do it:

Remove the fractions.

Multiply the brackets out.

Collect the letter terms on the left side and the number terms on the right.

Solve the equation.

## Example 2:

$$\frac{x+8}{2} + \frac{x+6}{4} = 7$$

$\times 2 \quad \times 4$  (ALL need to be multiplied)

$$\frac{\cancel{2} \times \cancel{4} \times (x+8)}{\cancel{2}} + \frac{\cancel{2} \times \cancel{4} \times (x+6)}{\cancel{4}} = 7 \times \cancel{2} \times \cancel{4}$$

$$4(x+8) + 2(x+6) = 56$$

$$4x + 32 + 2x + 12 = 56$$

$$6x = 12$$

$$\div 6$$

$$x = 2$$

For more help and resources, or  
to work with us as a tutor, please  
contact us

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