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EB Education Revision Guide

How to Work with Algebra: Part 4

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What you need to know about Algebra: Part 4

- Factorising quadratics

- Solving quadratics

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Factorising Quadratics

How to do it: When a=1

Quadratic equations are in the format

$$ax^2 + bx + c = 0$$

The name Quadratic comes from "quad" meaning square, because the variable gets squared (like x^2).

a, b and c are known values. In quadratic equations, a cannot be 0 because if it was it wouldn't be a quadratic.

x is the unknown value or variable.

Factorising quadratic equations means putting it back into two brackets.

This is easier to do when a = 1

$$ax^2 + bx + c$$

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Factorising Quadratics

How to do it:

Step 1: Make sure it is in the standard format.

$$ax^2 + bx + c = 0$$

Step 2: Write out your brackets.

$$(x \quad)(x \quad)$$

Step 3: Find two numbers that multiply together to make c (in example 1 it would be 9) and add together to make b (in this case 6).

Do this by writing down all the factor pairs of 9 (remember negative numbers as well).

Step 4: Choose the factor pair that will also add/subtract to make 6 – check the +/- signs!

Step 5: Put the numbers in the brackets.

Step 6: To check your answer. Expand the brackets out and make sure you get the original equation.

Example 1:

Factorise $x^2 + 6x + 9$

$(x \quad)(x \quad)$

3 and 3 multiply to make 9

3 and 3 add to make +6

$= (x + 3)(x + 3)$

$(x + 3)(x + 3) = x^2 + 6x + 9$

Factor pairs of 9

+ 1	+9
-1	-9
+3	+3
-3	-3

TOP TIP: Check out our guide "How to work with Algebra Part 2" for a reminder of how to expand double brackets

Factorising Quadratics

Example 2:

$x^2 - 12x + 20$

$(x \quad)(x \quad)$

-10 and -2 multiply to make 20

-10 and -2 add to make -12

$= (x - 10)(x - 2)$

Factor pairs of 20	
+1	+20
-1	-20
+5	+4
-5	- 4
+10	+2
-10	- 2

Example 3:

$x^2 - 6x - 40$

$(x \quad)(x \quad)$

-10 and +4 multiply to make -40

-10 and +4 add to make -6

$= (x - 10)(x + 4)$

Factor pairs of -40	
+1	-40
-1	+40
+5	-8
-5	+8
+10	-4
-10	-+4
+20	-2
-20	+2

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Factorising Quadratics

How to do it: When a is not 1

It gets a little trickier when a is not 1.

Step 1: Find two numbers that multiply together to give ac, and add together to give b.

Step 2: Rewrite the middle of the equation (7*x*) using those numbers.

Step 3: Factorise the first term.

Step 4: Factorise the second term.

Step 5: If you have done this correctly – you will have a common term in the brackets – you can remove one of these, and put the terms outside into the first bracket.

Step 6: Expand it back out to check your answer.

Example 1:

$3x^2 + 7x - 6$

$3x^2 + 9x - 2x - 6$

$3x^2 + 9x = 3x(x + 3)$

$-2x - 6 = -2(x + 3)$

$3x(x + 3)\ -2\ (x + 3)$

$= (3x - 2)(x + 3)$

Factor pairs of -18

(3 x - 6 = -18)

+1	-18
-1	+18
+6	-3
-3	-+6
+9	-2
-9	+2

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Factorising Quadratics

Example 2:

$2x^2 - 5x - 12$

$2x^2 - 8x + 3x - 12$

$2x^2 - 8x = 2x(x - 4)$

$3x - 12 = 3(x - 4)$

$2x(x - 4) + 3(x - 4)$

$(2x + 3)(x - 4)$

Factor pairs of - 24

(2 x - 12 = - 24)

+1	-24
-1	+24
+6	-4
-4	-+6
+8	-3
-8	+3
12	-2
-12	+2

Example 3:

$6x^2 + 5x - 6$

$6x^2 - 4x + 9x - 6$

$6x^2 - 4x = 2x(3x - 2)$

$9x - 6 = 3(3x - 2)$

$2x(3x - 2) + 3(3x - 2)$

$(2x + 3)(3x - 2)$

Factor pairs of - 36

(6 x -6 = - 36)

+1	-36
-1	+36
+6	-6
-6	-+6
+9	-4
-4	+9
12	-3
-12	+3
-13	+ 2
+13	-2

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Factorising Quadratics

TOP TIP: when c is positive

$$ax^2 + bx + c = 0$$

If **c is positive** then both signs in the brackets will be the **same**.

Look at b to decide if the numbers are both positive or both negative numbers.

When +bx	BOTH SIGNS IN BRACKETS ARE +
When – bx	BOTH SIGNS IN BRACKETS ARE -

TOP TIP: when c is negative

$$ax^2 + bx - c = 0$$

If **c is negative** then the signs in the two brackets will be **different**.

Look at b to determine which number is positive and which number is negative.

When + bx	THE SIGN IN THE BRACKET WITH THE LARGEST NUMBER IS +
When – bx	THE SIGN IN THE BRACKET WITH THE LARGEST NUMBER IS -

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Solving Quadratics

How to do it:

To solve the equation – you need to factorise first.

Then you solve it by setting each bracket equal to 0 because at least one of the brackets must be equal to 0.

i.e. (zero)(30)= 0

Or (30)(zero)= 0 or (zero)(zero)=0

Example 1:

Solve $x^2 - 12x + 20 = 0$

$= (x - 10)(x - 2)$

$(x - 10) = 0$ so $x = 10$

OR

$(x - 2) = 0$ so $x = 2$

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Solving Quadratics

Example 2:

Solve $x^2 - 6x - 40 = 0$

$= (x - 10)(x + 4)$

$(x - 10) = 0$ so $x = 10$

OR

$(x + 4) = 0$ so $x = -4$

Example 3:

Solve $3x^2 + 7x - 6 = 0$

$= (3x - 2)(x + 3)$

$(3x - 2) = 0$ so $x = \frac{2}{3}$

OR

$(x + 3) = 0$ so $x = -3$

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Your turn:

Set 1:

Factorise:

1. $x^2 + 7x + 10$
2. $x^2 + 8x + 15$
3. $x^2 - 14x + 40$
4. $x^2 - 10x + 21$
5. $x^2 - 7x - 18$
6. $x^2 - 6x - 7$

Set 2:

Factorise:

1. $x^2 + 5x - 24$
2. $x^2 - 8x + 12$
3. $x^2 + 3x - 40$
4. $x^2 - 9x + 20$
5. $x^2 - 13x - 30$
6. $x^2 + 2x - 35$

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Your turn:

Set 1:

Factorise:

1. $2x^2 + 5x - 3$
2. $2x^2 + 7x - 15$
3. $2x^2 - 9x + 4$
4. $2x^2 + 5x + 2$
5. $3x^2 + 10x - 8$
6. $6x^2 - 11x - 10$

Set 2:

Solve these equations:

1. $x^2 - 9x + 20 = 0$
2. $x^2 - 8x + 15 = 0$
3. $x^2 - 3x - 10 = 0$
4. $2x^2 + 5x = 12$

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Answers:

Set 1:

1. $(x + 5)(x + 2)$
2. $(x + 5)(x + 3)$
3. $(x - 10)(x - 4)$
4. $(x - 7)(x - 3)$
5. $(x - 9)(x + 2)$
6. $(x - 7)(x + 1)$

Set 2:

1. $(x - 3)(x + 8)$
2. $(x - 6)(x - 2)$
3. $(x - 5)(x + 8)$
4. $(x - 5)(x - 4)$
5. $(x - 15)(x + 2)$
6. $(x + 7)(x - 5)$

Remember: The brackets can be either way around i.e. (x-3)(x+4) gives the same as (x+4)(x-3)

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Answers:

Set 1:

1. $(2x - 1)(x + 3)$
2. $(2x - 3)(x + 5)$
3. $(2x - 1)(x - 4)$
4. $(2x + 1)(x + 2)$
5. $(3x - 2)(x + 4)$
6. $(2x - 5)(3x + 2)$

Set 2:

1. $(x - 5)(x - 4)$ so $x = +5$ or $+4$
2. $(x - 5)(x - 3)$ so $x = +5$ or $+3$
3. $(x - 5)(x + 2)$ so $x = +5$ or -2
4. $(2x - 3)(x + 4)$

   so $x = -4$ or $\frac{3}{2}$

Remember: The brackets can be either way around i.e. (*x*-3)(*x*+4) gives the same as (*x*+4)(*x*-3)

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For more help and resources, or to work with us as a tutor, please contact us

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