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EB Education Revision Guide

How to work with Sequences: Part 1

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Sequences

What is a sequence?

Number sequences are sets of numbers that follow **a pattern or a rule.**

- If the rule is to add or subtract a number each time, it is called an **arithmetic** sequence.
- If the rule is to multiply or divide by a number each time, it is called a **geometric** sequence.

Each number in a sequence is called a **term**.

A sequence which increases or decreases by the same amount each time is called a **linear sequence**.

Examples

Arithmetic sequences:

1, 5, 9, 13, 17 4 is added each time

12, 7, 2, –3, –8, 5 is subtracted each time

Geometric sequence:

1, 2, 4, 8, 16, 32 The number has been doubled each time

Number sequences

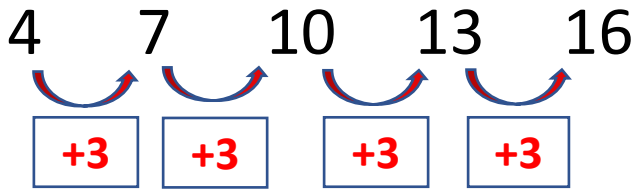
How to find the rule for number sequences?

You need to work out what you have to do to get from one number to the next.

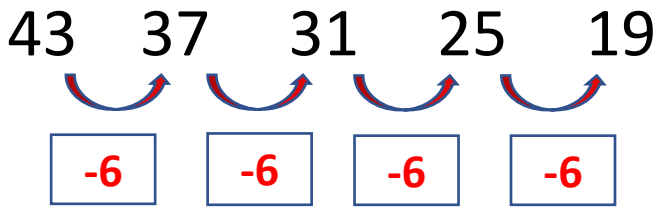
Write down what you do in the gaps between the numbers.

Examples

- Arithmetic sequences



Rule: Add 3 to the previous term



Rule: Subtract 6 from the previous term

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Number sequences

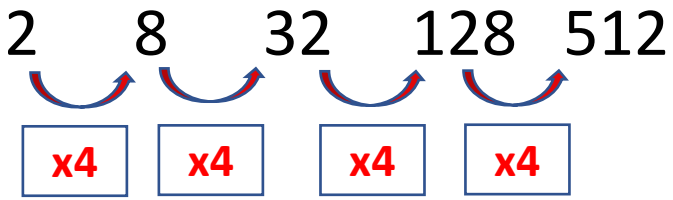
How to find the rule for number sequences?

You need to work out what you have to do to get from one number to the next.

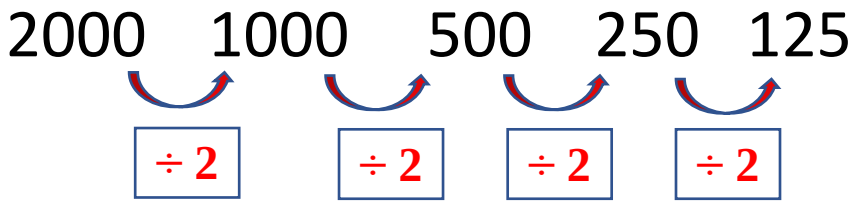
Write down what you do in the gaps between the numbers.

Examples

- Geometric sequences



Rule: Multiply the previous term by 4



Rule: Divide the previous term by 2

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Number sequences

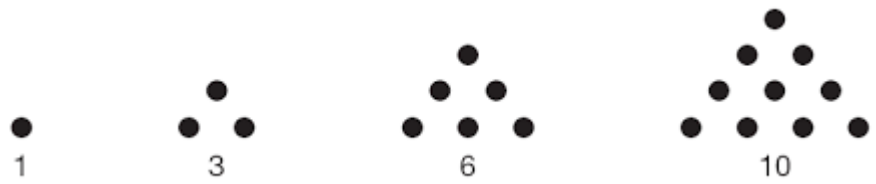
Triangular number sequence

1 3 6 10 15 21

+2 +3 +4 +5 +6

Rule: The number you add on increases each time

This is a triangular number sequence, as all the numbers are triangular. Triangular numbers can make a triangular dot pattern.



Fibonacci sequence

1 1 2 3 5 8 13

Rule: The next number is found by adding the two numbers before it.

1 + 1 = 2

1 + 2 = 3

2 + 3 = 5

This is known as the Fibonacci sequence.

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The n^{th} term

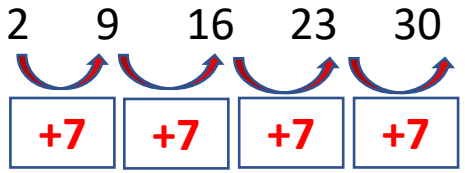
Finding the nth term of a Linear Sequence

Each term in a sequence has a position. The first term is in position 1, the second term is in position 2 and so on.

Position to terms rules use algebra to work out what number is in a sequence if the position in the sequence is known.

This is also called the n^{th} term, which is a position to term rule that works out a term at position 'n', where n means any position in the sequence.

Example



Step 1: Find the common difference between each number in the sequence.

It is +7. This means 7n is in the formula.

Step 2: As you are adding 7 each time- write the 7 times table (or values of 7n) and work out what you need to add or subtract to get from this number to the term in the sequence.

7	14	21	28	35
2	9	16	23	30

To get from 7n to the term is -5

Step 3: Put 7n and -5 together. The n^{th} term is 7n-5

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The n^{th} term

Finding the n^{th} term of a Linear Sequence (another method)

This can also be done more algebraically by using the formula:

n^{th} term = (common difference x n) + $Zero^{th}$ term

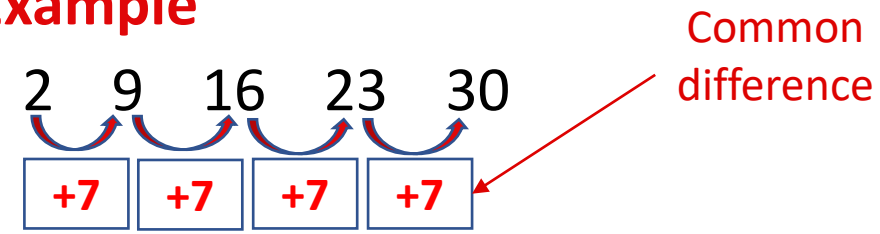
The common difference is the number that you are always adding or subtracting to each term

The n is the term position

The $Zero^{th}$ term is the term that comes before the first term in the given sequence

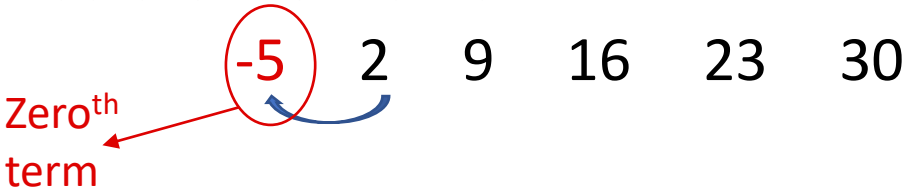
Either method works so just use the one you prefer!

Example



Step 1: Find the common difference between each number in the sequence.

Step 2: Find the $Zero^{th}$ term, i.e. the number before the first term



Step 3: put this into the formula

nth term = 7n -5

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The n^{th} term

How do you decide if a term is in a sequence?

If you are provided with the n^{th} term – you may be asked to state what a certain term is in that sequence or work out if a certain value is in that sequence.

To work this out, you need to put it into an equation, and solve it.

Example

What is the 16^{th} term of this sequence?

$4n + 2$

n = 16 as you are working out the 16^{th} term.
Substitute 16 into the nth term expression.

$(4 \times 16) + 2 = 66$

Is 164 a term in this sequence?

Set the expression as equal to 164, then solve it.

$$4n + 2 = 164$$
$$-2 \quad -2$$
$$4n = 162$$
$$\div 4 \quad \div 4$$
$$n = 40.5$$

n is not a whole number, so 164 is not in the sequence 4n + 2

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Your turn:

1. The first 5 terms of an arithmetic sequence are:

6 11 16 21 26

Find an expression, in terms of n for the n^{th} term of the sequence.

2. The first 5 terms of a number sequence are:

3 8 13 18 23

a) Write down the next two terms of the sequence.

b) Explain how you found your answer.

c) Explain why 387 is not a term of the sequence.

3. Here are the first five terms of a number sequence.

126 122 118 114 110

a) Write down the next two terms of the number sequence.

b) Explain how you found your answer.

c) The 20^{th} term of the number sequence is 50
What is the 21^{st} term of the number sequence.

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Your turn:

4. What is the next term for the following sequences

a) 5 9.5 14 18.5 ___

b) 22 18 14 10 ___

c) $\frac{3}{8}$ $\frac{5}{11}$ $\frac{7}{14}$ $\frac{9}{17}$ $\frac{11}{20}$

5. Which of these sequences is a geometric progression?

1 2 3 4 1 2 4 7

1 2 4 8 1 2 3 5

6. Look at the sequence below.

4 8 12 16 ___

The n^{th} term is 4n

a) What is the n^{th} term of the sequence below.

6 10 14 18

4n 6n 4n + 2 6n + 2

b) These points are in a straight line.

Point 1	(4, 6)
Point 2	(8,10)
Point 3	(12,14)
Point 4	(16,18)

Write down the co-ordinates of Point n in this sequence.

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Your turn:

7. The n^{th} term of a sequence is $2n + 1$
   The n^{th} term of a different sequence is $3n – 1$

Work out the three numbers that are:

- in both sequences
- between 20 and 40

8 a) Below are the fourth and fifth terms of a Fibonacci-type sequence.

____ ____ ____ 28 43

Show that the first term is 2.

b) Below are the first and third terms of a different Fibonacci-type sequence.

a ____ b ____ _____

Work out an expression in terms of a and b or the fifth term.

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Your turn:

9. Below are the first 5 terms of an arithmetic sequence.

2 9 16 23 30

a) What is the 12^{th} term of this sequence?

b) Find, in terms of n, an expression for the n^{th} term of this sequence.

10. Below are the first four terms of an arithmetic sequence.

21 17 13 9

Find an expression for the n^{th} term of this sequence.

11. Below are the first 5 terms of an arithmetic sequence.

6 11 16 21 26

Find an expression, in terms of n, for the nth term of the sequence.

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Answers:

1. The first 5 terms of an arithmetic sequence are:

6 11 16 21 26

Find an expression, in terms of n for the n^{th} term of the sequence.

5n +1

2. The first 5 terms of a number sequence are:

3 8 13 18 23

a) Write down the next two terms of the sequence.

28 33

b) Explain how you found your answer.

Add 5 each time

c) Explain why 387 is not a term of the sequence.

All terms end in 3 or 8 because you are adding 5 each time

3. Here are the first five terms of a number sequence.

126 122 118 114 110

a) Write down the next two terms of the number sequence.

106 102

b) Explain how you found your answer.

Subtract 4 each time

c) The 20^{th} term of the number sequence is 50.
What is the 21^{st} term of the number sequence.

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Answers:

4. What is the next term for the following sequences

a) 5 9.5 14 18.5 ___ 23 (adding 4.5 each time)

b) 22 18 14 10 ___ 6 (subtracting 4 each time)

c) $\frac{3}{8}$ $\frac{5}{11}$ $\frac{7}{14}$ $\frac{9}{17}$ $\frac{11}{20}$ $\frac{13}{23}$ (adding 2 to the numerator and adding 3 to the denomiator each time)

5. Which of these sequences is a geometric progression?

1 2 3 4

1 2 4 7

1 2 4 8

1 2 3 5

6. Look at the sequence below.

4 8 12 16 ___

The n^{th} term is 4n

a) What is the n^{th} term of the sequence below.

6 10 14 18

4n 6n 4n + 2 6n + 2

b) These points are in a straight line.

Point 1	(4, 6)
Point 2	(8, 10)
Point 3	(12, 14)
Point 4	(16, 18)

Write down the co-ordinates, of Point n in this sequence.

(4n, 4n +2)

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Answers:

7. The n^{th} term of a sequence is $2n + 1$
   The n^{th} term of a different sequence is $3n – 1$

Work out the three numbers that are:

- in both sequences 23, 29, 35
- between 20 and 40

2n + 1:
2 x 10 + 1 = 21
2 x 11 + 1 = 23
2 x 12 + 1 = 25
2 x 13 + 1 = 27
2 x 14 + 1 = 29
2 x 15 + 1 = 31
2 x 16 + 1 = 33
2 x 17 + 1 = 35
2 x 18 + 1 = 37
2 x 19 + 1 = 39

3n-1:
3 x 13 – 1 = 38
3 x 12 – 1 = 35
3 x 11 -1 = 32
3 x 10 - 1 = 29
3 x 9 - 1 = 26
3 x 8 - 1 = 23
3 x 7 - 1 = 20

8a) Below are the fourth and fifth terms of a Fibonacci-type sequence.

2 13 15 28 43

Show that the first term is 2.

28 + 15 = 43, 13 + 15 = 28 , 2 + 13 = 15

b) Below are the first and third terms of a different Fibonacci-type sequence.

a b-a b 2b -a 3b -a

Work out an expression in terms of a and b for the fifth term.

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Answers:

9. Below are the first 5 terms of an arithmetic sequence.

7	14	21	28	35
2	9	16	23	30

a) What is the 12th term of this sequence?

(7 x 12) – 5 = 79

b) Find, in terms of n, an expression for the nth term of this sequence.

+ 7 each time, so it is 7n
Write the 7 x table above
To get from 7 to 2, 14 to 9 etc
you subtract 5

7n -5

10. Below are the first four terms of an arithmetic sequence.

-4	-8	-12	-16
21	17	13	9

Find an expression for the nth term of this sequence.

-4 each time, so it is -4n
Write the -4 x table above
To get from -4 to 21, -8 to 17,
you add 25

-4n +25

11. Below are the first 5 terms of an arithmetic sequence.

6 11 16 21 26

Find an expression, in terms of n, for the nth term of the sequence.

5n + 1

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For more help and resources, or to work with us as a tutor, please contact us

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