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EB Education Revision Guide

How to work with Sequences: Part 2

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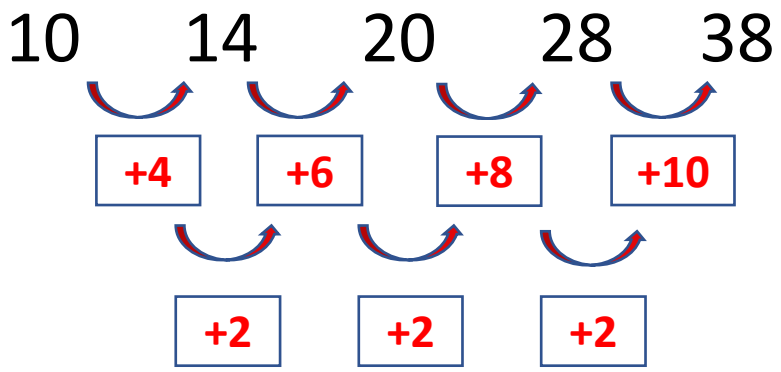
Quadratic Sequences

What is a quadratic sequence?

Quadratic sequences are sequences which contain an n^2 term.

In a quadratic sequence the differences between the terms are not equal, but the second differences (i.e. the difference between the differences) between the terms are equal.

Examples



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Quadratic sequences

How to find terms in a quadratic sequence?

To work out terms in a quadratic sequence, you follow the same rules as you would for a linear sequence.

You need to substitute the value of n into the formula.

Examples

Find the first 5 terms of $n^2 + 3n - 5$

- When n = 1

$1^2 + (3 \times 1) - 5 = -1$

- When n = 2

$2^2 + (3 \times 2) - 5 = 5$

- When n = 3

$3^2 + (3 \times 3) - 5 = 13$

- When n = 4

$4^2 + (3 \times 4) - 5 = 23$

- When n = 5

$5^2 + (3 \times 5) - 5 = 35$

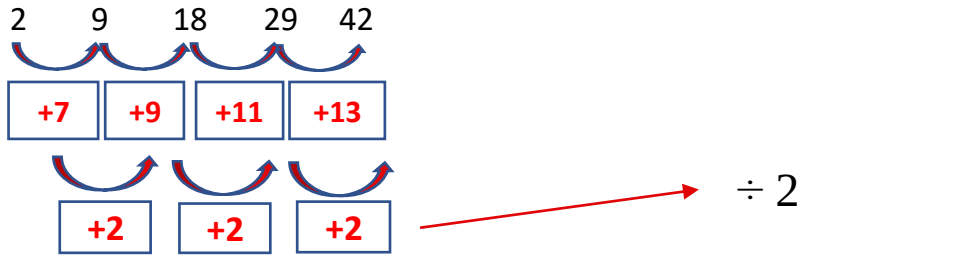
The first 5 terms are:
-1, 5, 13, 23, 35

The n^{th} Term

Finding the n^{th} Term of a Quadratic Sequence (Method 1)

- Find the difference between each pair of terms.
- As the difference is changing each time, then find the difference between the differences.
- Divide this difference by 2. This provides you with the coefficient (the number in front) of the n^2 term.
- Subtract the n^2 term from each term in the sequence. You will be left with a linear sequence.
- Find the rule for the n^{th} term of the linear sequence.
- Put the n^2 term and the n^{th} term together by adding them

Example 1



This means there is an n^2 term.

n:	1	2	3	4	5
Term:	2	9	18	29	42
n^2	1	4	9	16	25
Term – n^2:	1	5	9	13	17

+4 +4 +4 +4

4	8	12	16	-3
1	5	9	13	

n^{th} term of the linear sequence is 4n-3

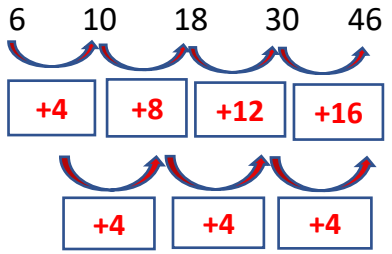
The expression for the n^{th} term is **$n^2 + 4n - 3$**

The n^{th} Term

Finding the n^{th} Term of a Quadratic Sequence (Method 1)

- Find the difference between each pair of terms.
- As the difference is changing each time, then find the difference between the differences.
- Divide this difference by 2. This provides you with the coefficient (the number in front) of the n^2 term.
- Subtract the n^2 term from each term in the sequence. You will be left with a linear sequence.
- Find the rule for the n^{th} term of the linear sequence.
- Put the n^2 term and the n^{th} term together by adding them

Example 2



$\div 2$

This means there is a $2n^2$ term.

n:	1	2	3	4	5
Term:	6	10	18	30	46
$2n^2$	2	8	18	32	50
Term – $2n^2$:	4	2	0	-2	-4
	-2	-2	-2	-2	

-2 -4 -6 -8
4 2 0 -2
+6

n^{th} term of the linear sequence is -2n + 6

The expression for the n^{th} term is **$2n^2 - 2n + 6$**

The n^{th} Term

Finding the n^{th} Term of a Quadratic Sequence (Method 2)

The n^{th} term of the quadratic sequence will be of the form:

$$an^2 + bn + c$$

You can use a formula for working out a, b and c.

Formulas to use:

$2a = 2^{nd}$ difference (this is always constant)

$3a + b = 2^{nd}$ term $- 1^{st}$ term

$a + b + c = 1^{st}$ term

	n = 1	n = 2	n = 3
Term	a + b + c	4a + 2b + c	9a + 3b + c
1^{st} difference		3a + b	5a + 5
2^{nd} difference		2a	

Substitute the value of n into the formula

e.g. when n=3 → $a(3)^2 + b(3) + c$

$9a + 3b + c$

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Formulas to use:

$2a = 2^{nd}$ difference (this is always constant)

$3a + b = 2^{nd}$ term $- 1^{st}$ term

$a + b + c = 1^{st}$ term

$an^2 + bn + c$

The n^{th} Term

Example

- Find the differences until you have a common difference

- Use the first formula

$2a = 2^{nd}$ difference (common difference)

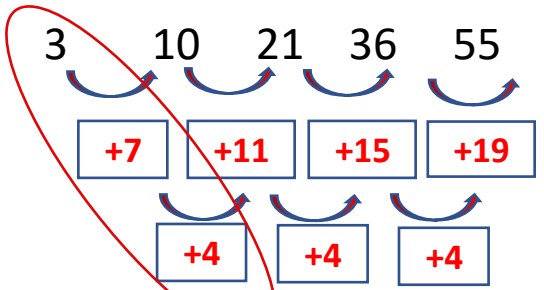
- Once you have worked out the value of a, use the second formula

$3a + b = 2^{nd}$ term $- 1^{st}$ term i.e. $10 - 3 = 7$

- Once you have worked out a and b, use the third formula

$a + b + c = 1^{st}$ term

Find the n^{th} term of this sequence:



Use the numbers in these positions with the formulae

$2a = +4$ so $a = 2$

$3a + b = 7$

$(3 \times 2) + b = 7$

$6 + b = 7$

$b = 7 - 6$

So $b = 1$

$a + b + c = 3$

$2 + 1 + c = 3$

$3 + c = 3$

So $c = 0$

Put together: $2n^2 + n$

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Your turn:

1. What are the next two terms in the quadratic sequence below.

   9 13 19 27

2. What are the next two terms in the quadratic sequence below.

   -5 0 9 22

3. The n^{th} term of a sequence is

   $2n^2 + 4n - 1$

   Work out the 10^{th} term of the sequence.

4. The n^{th} term of a sequence is

   $n^2 + 2n$

   Work out the first 5 terms in this sequence.

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Your turn:

5. Work out the formula for the n^{th} term of the following quadratic sequence:

5 11 19 29

6. Work out the formula for the n^{th} term of the following quadratic sequence:

2 10 22 38

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Your turn:

7. Work out the formula for the n^{th} term of the following quadratic sequence:

19 15 9 1

8 a) Look at the below sequence.

6 10 16 24

Show that the n^{th} term is $n^2 + n + 4$

b) Hence find the term that has value 136

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Your turn:

9. Work out the formula for the n^{th} term of the quadratic sequence:

19, 15, 9, 1…..

10. A quadratic sequence begins:

6, 10, 16, 24……

a) Show that the n^{th} term is $n^2 + 2 + 4$

b) Find the term that has value 136

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Answers:

1. What are the next two terms in the quadratic sequence below.

9 13 19 27

+ 4 + 6 + 8

So next 2 terms will be + 10 + 12

37 49

2. What are the next two terms in the quadratic sequence below.

-5 0 9 22

+ 5 + 9 + 13

So next 2 terms will be + 17 + 21

39 60

3. The n^{th} term of a sequence is

$2n^2 + 4n - 1$

Work out the 10^{th} term of the sequence.

$(2 \times 10^2) + (4 \times 10) - 1$

$200 + 40 - 1 = 239$

4. The n^{th} term of a sequence is

$n^2 + 2n$

Work out the first 5 terms in this sequence.

$1^2 + 2 = 3$ $2^2 + 4 = 8$

$3^2 + 6 = 15$ $4^2 + 8 = 24$

$5^2 + 10 = 35$

Answers:

5. Work out the formula for the n^{th} term of the following quadratic sequence:

5 11 19 29

+ 6 + 8 + 10

+ 2 +2 ÷ 2 = $\mathbf{n^2}$

Term	5	11	19	29
n^2	1	4	9	16
Term-n^2	4	7	10	13
	+3	+ 3	+3	= **3n**

3 6 9 12

4 7 10 13 **+1**

n^2 + 3n +1

6. Work out the formula for the n^{th} term of the following quadratic sequence:

2 10 22 38

+8 + 12 +16

+ 4 +4 ÷ 2 = $\mathbf{2n^2}$

Term	2	10	22	38
$2n^2$	2	8	18	32
Term-$2n^2$	0	2	4	6
	+2	+2	+2	= **2n**

2 4 6 8

0 2 4 6 **-2**

$2n^2$ + 2n -2

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Answers:

7. Work out the formula for the n^{th} term of the following quadratic sequence:

19 15 9 1

-4 -6 -8

-2 -2 $\div 2 = \mathbf{-n^2}$

- and – make a +

Term	19	15	9	1
$-n^2$	-1	-4	-9	-16
Term- $-n^2$	20	19	18	17

-1 -1 -1 $= \mathbf{-n}$

-1 -2 -3 -4

20 19 18 17 **+21**

$\mathbf{-n^2 - n + 21}$

TOP TIP:
Remember how to solve a quadratic equation. Look at our other " How to guides" to help you, like our solving linear sequences guide

8 a) Look at the below sequence.

6 10 16 24

Show that the n^{th} term is $n^2 + n + 4$

+4 +6 +8

+2 +2 $\div 2 = \mathbf{n^2}$

Term	6	10	16	24
n^2	1	4	9	16
Term-n^2	5	6	7	8

+1 +1 +1 $= \mathbf{n}$

1 2 3 4

5 6 7 8 **+4**

$\mathbf{n^2 + n + 4}$

b) Hence find the term that has value 136

$n^2 + n + 4 = 136$

$n^2 + n - 132 = 0$ $(n + 12)(n - 11) = 0$

$n = -12$ or $n = 11$

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Answers:

9. Work out the formula for the n^{th} term of the quadratic sequence:

19, 15, 9, 1…..

-4 -6 -8

-2 -2 ÷ 2 = **$-n^2$**

Term	19	15	9	1
$-n^2$	-1	-4	-9	-16
Term- $-n^2$	20	19	18	17

-1 -1 -1 = **-n**

-1 -2 -3 -4

20 19 18 17 **+21**

$-n^2 - n + 21$

- and – make a +

TOP TIP:
Remember how to solve a quadratic equation. Look at our other " How to guides" to help you

10. A quadratic sequence begins:

6, 10, 16, 24……

a) Show that the n^{th} term is $n^2 + 2 + 4$

+4 +6 +8

+2 +2 ÷ 2 = **n^2**

Term	6	10	16	24
n^2	1	4	9	16
Term-n^2	5	6	7	8

+1 +1 +1 = **n**

1 2 3 4

5 6 7 8 **$n^2 + n + 4$**

b) Find the term that has value 136

$n^2 + n + 4 = 136$

$n^2 + n - 132 = 0$ $(n + 12)(n - 11) = 0$

$n = -12$ or $n = 11$

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For more help and resources, or to work with us as a tutor, please contact us

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